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Charged Tachyons and Gauge Symmetry Breaking

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Abstract

We discuss the condensation of charged tachyons in the heterotic theory on the Kaluza-Klein Melvin background. The arguments are based on duality relations which are expected to hold from the adiabatic argument. It is argued that in many cases the rank of the gauge group is not changed by the condensation, as opposed to naive expectations.

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1 Introduction

To discuss properties of a theory, one has to find a stable vacuum of the theory. Thus it is important to investigate the (in)stability of vacua of the theory. In string theory, there are infinitely many perturbative vacua, and some of them are obviously unstable since the theories on such vacua contain tachyons in their mass spectra. In general, tachyons are not always excluded by the consistency conditions of the worldsheet theory, and in fact, there are many non-supersymmetric theories with tachyons. Thus it is necessary to consider how to understand such vacua.

String theories which contain tachyons may look strange. However, recent research reveals that tachyons in the open string sector can be regarded as something like Higgs bosons [1] (See also e.g. [2] and references therein). There is a nontrivial potential for the tachyons, and the tachyonic instability in the original vacuum is just due to the fact that the theory is defined around a maximum of the potential. The potential has global minima which correspond to stable vacua. These vacua are understood as states in which the tachyons condense and have nonzero vevs. The open string tachyons appear on unstable D-brane systems, and the tachyon condensation describes the decay of such D-brane systems. An important lesson we have learned from the open string tachyon condensation is that the existence of tachyons would be just a signal of an instability, not the inconsistency of the theory.

It would be expected that tachyonic vacua in closed string theories can also be understood in the same spirit. An important step toward this direction was made by making a conjecture which relates the tachyonic instability of Type 0A theory and the instability of the Kaluza-Klein Melvin background [3] (below we will call it the KK-Melvin, for short). The KK-Melvin is a “twisted” compactification of the flat spacetime on S^1 . String theories on the KK-Melvin have been studied in [4]. It was shown that the KK-Melvin is stabilized by an instanton effect [5][6] which leads the KK-Melvin to the supersymmetric background, i.e. $\mathbf{R}^{1,n} \times S^1$. This leads one to the expectation that Type 0A theory would have a stable ground state which corresponds to Type IIA theory with the maximal supersymmetry restored. Similar argument was applied to the non-supersymmetric heterotic theories [7][8]. There are also related works on the KK-Melvin, its generalizations and string theories on them [9]-[27].

There are other non-supersymmetric backgrounds of string theory, i.e. non-supersymmetric orbifolds which are studied in the context of the tachyon condensation. They were studied in [28], and more deeply investigated recently in [29][30] from the point of view of the worldsheet theory. Moreover, there is a proposal for the tachyon potential for closed string tachyons [31].

In this paper, we would like to investigate the heterotic theory on the KK-Melvin, by using duality relations to other theories, which was briefly discussed in [8]. Our main interest is the gauge symmetry breaking induced by the condensation of charged tachyons: In many cases, there are tachyons which form a nontrivial representation of the gauge group. As argued in [18], the condensation of such tachyons would break the gauge

symmetry and reduce the rank of the gauge group. On the other hand, the rank is severely restricted for the theory to be consistent. For example, in the supersymmetric heterotic theories in ten dimensions, the rank must equal to sixteen to cancel the gauge and the gravitational anomaly. Therefore one might worry that the tachyon condensation would spoil the consistency of the theory. We will argue that at least in some situations it is not the case, and show how such disaster can be avoided.

The organization of this paper is as follows. In section 2, we discuss duality relations of string theories on the KK-Melvin, based on the adiabatic argument [32]. The appearance of tachyonic modes in the heterotic theories on the KK-Melvin is explicitly shown in section 3. We consider a dual picture in M-theory in section 4, and find an interesting relation between the tachyonic instability and the dielectric effect [33]. We discuss, in section 5, dual string theories for several situations to deduce the fate of the gauge symmetry after the tachyon condensation. Section 6 is devoted to discussions.

2 String duality on Melvin background

The duality relations among all the five string theories and the eleven-dimensional supergravity enable us to investigate many nonperturbative aspects of the theories [34]. The strong coupling behavior of a theory can be discussed by considering the weak coupling behavior of its dual theory. There are many pieces of nontrivial evidence for the existence of the dual pairs. However, the arguments heavily depend on the existence of the extended supersymmetry, and it becomes hard to show such duality relations when there are less number of supersymmetries and, of course, when there is no supersymmetry at all.

There is a nice argument [32] which enables one to produce a new dual pair from a known one. The prescription is as follows: Suppose that a theory A and a theory B are dual to each other. Consider a symmetry h_A of A . The duality relation ensures that the corresponding symmetry h_B exists in B . In general, the orbifold A/h_A is not dual to the orbifold B/h_B . In fact, a counterexample for such an expectation is shown in [32]. A dual pair can be obtained by first compactifying both theories on, for example, S^1 , and then orbifolding them by $h_{A(B)}\sigma_{1/2}$, where $\sigma_{1/2}$ is the shift operator along the S^1 by a half circumference. That is, the theory A on S^1 twisted by $h_A\sigma_{1/2}$ is dual to the theory B on S^1 twisted by $h_B\sigma_{1/2}$. To see whether the duality relation holds, let the radius of the S^1 be very large. Then the effect of the orbifolding will be negligible and the local physics of each orbifold is governed by the original theory A or B . Thus the orbifolds are dual to each other at least locally. Moreover, they are dual to each other globally, since they can be understood as an example of a pair related by the fiberwise duality. An important point of the above construction of dual pairs is that the discrete group $(h_{A(B)}\sigma_{1/2}$ in the above case) acts freely. In fact, h_A alone does not always act freely on A , and the simple orbifold B/h_B cannot always be the dual of A/h_A .

This prescription has been employed to obtain dual pairs with less number of supersymmetries [35]. See also [36]. Moreover, it has also been applied to the cases in which there is no supersymmetry [37], to discuss properties of non-supersymmetric theories similar to the one in [38] to all orders in perturbation expansion as well as nonperturbatively.

In our previous paper [8], we have assumed a duality relation between Type I theory and the heterotic theory both on the KK-Melvin, to argue the strong coupling behavior of the latter theory, in spite of the absence of supersymmetry. This is based on a similar argument to the adiabatic argument explained above. Since the KK-Melvin is locally flat, the local physics of the former theory should be dual to that of the latter one. In addition, the duality transformation of the spacetime fields in the supergravity tells us that the KK-Melvin in the heterotic theory corresponds to the same KK-Melvin in Type I theory. Therefore the above two theories would be dual to each other.

Indeed, this is the case to which the adiabatic argument can be applied. It has been shown [7][8] that the heterotic theory on the KK-Melvin with special values of parameters (and some patterns of Wilson lines) can be reinterpreted as an orbifold.

To make the statement more definite, let us write down the metric of the KK-Melvin,

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + r^2(d\theta + qdy)^2 + dy^2, \quad (2.1)$$

where $\mu, \nu = 0, \dots, 6$. We have used the polar coordinates r, θ for the 7-8 plane. The y -direction is compactified on S^1 with the radius R . There is a real parameter q which represents the nontriviality of the global structure of the spacetime. The parameter q has a periodicity with the period $2/R$ when there exist spacetime fermions.

The one-loop partition function of the heterotic theory on the KK-Melvin can be obtained explicitly,

$$\begin{aligned} Z_q(A_Y^I) = & \int \frac{d^2\tau}{\tau_2} \tau_2^{-4} |\eta(\tau)|^{-12} \sum_{m,n \in \mathbf{Z}} \exp \left[-\frac{\pi R^2}{\alpha' \tau_2} |n + m\tau|^2 \right] |Z_{1+2nqR}^{1+2mqR}(\tau)|^{-2} \\ & \exp \left[\pi i m n \left\{ (qR)^2 - \sum_{I=1}^{16} (A_Y^I R)^2 \right\} \right] e^{2\pi i m q R} Z_L^{(m,n)}(\tau) Z_{1+nqR}^{1+mqR}(\tau)^{*4}, \quad (2.2) \end{aligned}$$

where A_Y^I is a Wilson line put on the y -direction. See [8] for the details of the expressions. This partition function can be rewritten as that of an orbifold, if one chooses $q = 1/R$ and an appropriate Wilson line. The orbifold is the heterotic theory on S^1 twisted by the operator $(-1)^{F_s} \gamma_\delta \sigma_{1/2}$, where F_s is the spacetime fermion number and γ_δ is a rigid gauge transformation which is determined by the choice of the Wilson line. This is a right orbifold theory to apply the adiabatic argument, and the dual theory should be the Type I theory on S^1 twisted by the operator $(-1)^{F_s} \gamma'_\delta \sigma_{1/2}$, where γ'_δ is the corresponding gauge transformation in Type I theory. One can easily see that the closed string sector of the dual orbifold is just that of Type I theory on the KK-Melvin with the same value of q . Therefore, since the consistency of the theory determines its open string sector, it is concluded that the dual of the heterotic theory on the KK-Melvin is Type I theory on the same KK-Melvin, as expected.

The above relation between the KK-Melvin and the freely-acting orbifold can be found without one-loop calculations. Consider the worldsheet action of the heterotic theory on the KK-Melvin with a Wilson line,

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \left\{ \eta_{\mu\nu} \partial_a X^\mu \partial_a X^\nu + |\partial_a X + iq \partial_a Y X|^2 + (\partial_a Y)^2 \right\} \\ + \frac{i}{\pi} \int d^2\sigma S^{r\dagger} \left(\partial_+ + \frac{i}{2} q \partial_+ Y \right) S^r + \frac{i}{\pi} \int d^2\sigma \sum_{I=1}^{16} \lambda^{I\dagger} \left(\partial_- - i A_Y^I \partial_- Y \right) \lambda^I, \quad (2.3)$$

where $r = 1, \dots, 4$. The right-moving fermions S^r are the Green-Schwarz fermions while the left-moving fermions λ^I are the RNS fermions. This action can be reduced to the free action by the field redefinitions

$$X = e^{-iqY} \tilde{X}, \\ S^r = e^{-\frac{i}{2}qY} \tilde{S}^r, \\ \lambda = e^{iA_Y^I Y} \tilde{\lambda}^I. \quad (2.4)$$

The new fields satisfy the twisted boundary conditions

$$Y(\sigma + 2\pi) = Y(\sigma) + 2\pi w R, \\ \tilde{X}(\sigma + 2\pi) = e^{2\pi i w q R} \tilde{X}(\sigma), \\ \tilde{S}^r(\sigma + 2\pi) = e^{\pi i w q R} \tilde{S}^r(\sigma), \\ \tilde{\lambda}(\sigma + 2\pi) = e^{-2\pi i w A_Y^I R} \tilde{\lambda}(\sigma). \quad (2.5)$$

By setting $qR = 1$ and choosing an appropriate Wilson line, one can see that this theory is equivalent to the orbifold mentioned above (winding sectors with odd w form the twisted sector). Note that the radius of the S^1 of the orbifold is twice as large as the original one. Similarly, one can also show the equivalence between the KK-Melvin and the orbifold even when qR is a rational number. Thus, according to the adiabatic argument, the Type I-heterotic duality also holds for this case. The equivalence to an orbifold for any rational qR is shown in terms of partition functions in Type IIA case [23].

When qR is an irrational number, the corresponding worldsheet theory has infinite number

of twisted sectors, and the equivalence would not make sense. However, naively it could be expected that the equivalence persists for any irrational qR . Suppose that strings live in a spacetime of the form $X^{1,6} \times \mathbf{R} \times \mathbf{R}^2$, where $X^{1,6}$ is a seven-dimensional spacetime. Denote this theory as C . Then the string theory on $X^{1,6} \times \text{Melvin}$ can be reinterpreted as the orbifold C/TS , where T is the translation along the \mathbf{R} and S is the spatial rotation in the \mathbf{R}^2 . The action of TS does not have any fixed point, and therefore the duality relation might exist for general value of qR .

Once the Type I-heterotic duality is assumed even when the theories live on the KK-Melvin, all the duality relations generated by it and the T-duality could be used to study the theories, according to the arguments in [36]. We will show below that the use of the

duality relations among string theories on the KK-Melvin is powerful enough to discuss what happens when closed string tachyons condense in the heterotic theory on the KK-Melvin.

3 Perturbative spectrum

In this section, we briefly show the perturbative spectrum of the heterotic theory on the KK-Melvin with a Wilson line. For definiteness, we choose the parameters as $qR = 1$ and

$$A_Y^1 R = (1, 0, \dots, 0). \quad (3.1)$$

This background gives the same spectrum as the non-supersymmetric $SO(32)$ heterotic theory in the limit $R \rightarrow \infty$ [7][8]. Note that in this limit, the adiabatic argument would not be applicable and investigations of the tachyon condensation in the theory based on duality arguments might not make sense. However, as we will see below, tachyons already appear for small but finite radius of the S^1 in the KK-Melvin, and hence the investigations of the tachyon condensation in the KK-Melvin can be carried out by using duality arguments.

To see the spectrum, it is convenient to employ RNS formalism for right-moving fermions. Their worldsheet action is

$$S_R = \frac{i}{\pi} \int d^2\sigma \left\{ \psi_\mu \partial_+ \psi^\mu + \psi^\dagger (\partial_+ + iq \partial_+ Y) \psi + \psi^Y \partial_+ \psi^Y \right\}. \quad (3.2)$$

The choice of the parameters above makes the perturbative calculations very easy. It is because the boundary conditions for the winding sectors do not change, while the GSO projections for both left- and right-movers are reversed. The mass operator and the level-matching condition are

$$\frac{\alpha'}{2} M^2 = \frac{\alpha'}{2} p^2 + \frac{(Rw)^2}{\alpha\alpha'} + N_L + N_R - E_0, \quad (3.3)$$

$$N_L - N_R = -wpR, \quad (3.4)$$

where $N_{L(R)}$ are the ordinary level operators and E_0 is the zero-point energy which is the same value as the flat background. The zero mode p of Y is expressed as follows,

$$p = \frac{1}{R} \left[m - \int_0^{2\pi} d\sigma \left\{ \frac{i}{4\pi\alpha'} (\partial_\tau \tilde{X}^\dagger \tilde{X} - \tilde{X}^\dagger \partial_\tau \tilde{X}) - \frac{1}{2\pi} \psi^\dagger \psi \right\} - \int_0^{2\pi} d\sigma \frac{1}{2\pi} \lambda^{1\dagger} \lambda^1 \right]. \quad (3.5)$$

Here \tilde{X} is defined by the eq.(2.4).

In the sector with $w = 0$, the spectrum is the same as that for the flat background. This is because, in this sector, the only difference from the flat case is the modification of p , and $pR - m$ is an integer for each state in the Fock space. Thus the shift of p due to the presence of the nontrivial background can be cancelled by shifting the KK-momentum number m .

In the winding sectors with odd w , the GSO projections for both the left- and the right-movers are reversed. One can see that tachyonic states exist only in the NS-NS sector. The lightest states in the sector with an odd w are

$$\begin{aligned} \tilde{\lambda}_{-\frac{1}{2}}^k |0\rangle_{NS-NS}, \quad \tilde{\lambda}_{-\frac{1}{2}}^{k\dagger} |0\rangle_{NS-NS}, \quad (k = 2, \dots, 16), \\ \tilde{\lambda}_{-\frac{1}{2}}^1 |m = -1\rangle_{NS-NS}, \quad \tilde{\lambda}_{-\frac{1}{2}}^{1\dagger} |m = 1\rangle_{NS-NS}, \end{aligned} \quad (3.6)$$

and their masses are

$$\frac{\alpha'}{2} M^2 = \frac{(wR)^2}{2\alpha'} - 1. \quad (3.7)$$

Therefore, tachyonic states appear in the theory if $R < \sqrt{2\alpha'}$. It is clear from the expressions of the states (3.6) that the tachyons couple to the gauge fields.

To see that the above 32 tachyonic states form a vector representation of the $SO(32)$ gauge group, it is convenient to consider the bosonic construction of the current algebra. For the supersymmetric $SO(32)$ theory, the internal momentum lattice is denoted as Γ_{16} , which is generated by the root lattice of $SO(32)$ and a weight

$$\left(\frac{1}{2}, \frac{1}{2}, \dots, \frac{1}{2}\right), \quad (3.8)$$

in a suitable basis. This means that the lattice contains the root lattice and one spinor lattice of $SO(32)$. The momentum lattice is shifted by turning on a Wilson line. Consider the Wilson line (3.1) used in the above calculation. Then the shifted lattice contains the vector lattice and one spinor lattice which has the opposite chirality to the one in the original lattice Γ_{16} . States in the winding sectors with odd w correspond to weights in the shifted lattice, and hence the tachyonic states form the vector representation.

Note that the tachyons with the winding number $w = +1$ and $w = -1$ have the same mass squared and they appear in the same range of R , i.e. $R < \sqrt{2\alpha'}$. As R becomes smaller, there appear more number of tachyons and the process of the tachyon condensation might become more complicated. We will restrict ourselves to the simplest case in which tachyons only appear in the $w = \pm 1$ sectors.

4 M-theory dual

It is well-known [39] that the heterotic theory compactified on T^3 is dual to M-theory compactified on K3. In this section, we assume that the duality between the heterotic

theory on $T^3 \times \text{Melvin}$ and M-theory on $K3 \times \text{Melvin}$ holds, and we discuss the dual picture of the tachyonic instability of the heterotic theory. Similar dualities are discussed recently [40].

As shown in the previous section, the tachyons in the heterotic theory have the winding number and the charges which couple to the gauge fields. One can find the dual states in M-theory by looking for states which have the corresponding quantum numbers. From the correspondence of spacetime fields between two supergravity theories, the quantum numbers are related as follows [39].

The heterotic theory on T^3 has the gauge group of the rank 22. For generic points of the moduli space, the gauge group is broken to $U(1)^{22}$. The corresponding gauge fields in M-theory come from the 3-form field via the Kaluza-Klein reduction. The number of the gauge fields is the same since the dimension of the second cohomology group $H^2(K3)$ is 22. Therefore the charged states in the heterotic theory would correspond to bound states which include M2-branes wrapped on some 2-cycles in the K3. Non-Abelian gauge groups appear when the K3 becomes singular [39]. The extra gauge fields (“W-bosons”) come from M2-branes wrapped on the vanishing 2-cycles at the singularity.

The NS-NS B-field couples to the winding number corresponding to the S^1 in the KK-Melvin. Its dual field in M-theory is the Hodge dual of the 3-form field. Therefore the winding states in the heterotic theory would correspond to bound states which include M5-branes wrapped on the K3 and the S^1 in the KK-Melvin.

Thus we conclude that the charged tachyons in the heterotic theory on the KK-Melvin would correspond to M2-M5 bound states in M-theory. Note that the instability we will discuss below is the one due to the bound states of branes, and hence the phenomena induced by them might be different from the ones induced by tachyonic strings argued in several papers [28][29][30][31].

It is convenient to see this situation in Type IIA picture, by the dimensional reduction along the S^1 in the KK-Melvin. This is because in this picture the instability can be understood in terms of the local physics, while in M-theory picture the instability originates in the global geometry. The resulting background in the Type IIA picture is known as a fluxbrane,

$$\begin{aligned} ds^2 &= \sqrt{1 + q^2 r^2} (\eta_{\alpha\beta} dx^\alpha dx^\beta + ds_{K3}^2 + dr^2) + \frac{r^2}{\sqrt{1 + q^2 r^2}} d\theta^2, \\ e^{\frac{4}{3}\Phi} &= 1 + q^2 r^2, \\ A_\theta &= \frac{qr^2}{1 + q^2 r^2}, \end{aligned} \tag{4.1}$$

where $\alpha, \beta = 0, \dots, 3$. In this picture, the corresponding states to the charged tachyons are D2-D4 bound states wrapped on the K3.

It is discussed [13][15] that in the presence of the fluxbrane D4-branes are blown up to a D6-brane whose worldvolume is $S^2 \times (\text{worldvolume of D4-branes})$. This phenomenon can be understood in terms of the dielectric effect of D-branes [33]. Suppose, for simplicity, that the D4-branes are at the center of the fluxbrane, i.e. at $r = 0$. Then, around the

D4-branes, the background is approximated by the geometry of the flat spacetime times K3, the constant dilaton, and the constant R-R field strength. In particular, the field strength is magnetic, so that it is equivalent to the 8-form electric R-R field strength. The non-Abelian version of the Chern-Simons terms in the worldvolume theory of D4-branes contains the coupling of the D4-branes to the 7-form R-R field. This interaction makes the spherical distribution of the D4-branes more stable, and the configuration locally has the D6-brane charge (although there is no net D6-brane charge). The solution for the dielectric D4-branes in the supergravity is obtained and its stability is examined [13][15].

We would like to relate the tachyonic instability in the heterotic theory to the instability of the D2-D4 systems in the fluxbrane against the blowing-up to spherical D2-D4-D6 systems. It is based on the following observations.

First of all, in the heterotic theory the tachyons exist only in the winding sectors, as shown in the previous section. The duality between the heterotic theory and M-theory would relate the winding number and the D4-brane charge. Thus it would be natural to relate the tachyonic instability to the one due to the presence of the D4-branes.

In [3], it is conjectured that Type 0A theory is dual to Type IIA theory with the fluxbrane background (4.1) with the ds^2_{K3} replaced with the flat one. In addition, it is argued that the tachyonic instability in Type 0A theory is related to the instability due to pair creations of D6-branes. See also [5][6]. Through the pair creation, it is argued that the fluxbrane would decay into the flat spacetime, so that in our case it would be expected that the blowing-up of the D2-D4 systems leads to a stable vacuum.

It is very interesting to investigate the potential for the radius ρ of the spherical D6-brane in the R-R background. The investigations of the gravity solution show that qualitative properties of the spherical D6-D4 system can be seen from the probe analysis [33][15], although the calculations have been performed in the flat background geometry which is not consistent. The potential is

$$V(\rho) \propto \sqrt{\rho^4 + a} - b\rho^3, \quad (4.2)$$

where a, b are constants and b is proportional to q . The shape of the potential $V(\rho)$ is shown in figure 1,2.

For large b , the potential has a unique extremum, i.e. maximum at $\rho = 0$ (figure 1). On the other hand, for small b there is a local minimum at a finite ρ (figure 2), and there is a potential barrier for the D6-brane to infinitely expand. It could be expected that $\rho \rightarrow \infty$ corresponds to the decay of the fluxbrane (or, in other words, the stabilization of the tachyonic instability). Consider the case with fixed qR and vary R . Then q is inversely proportional to the radius R of the S^1 in the KK-Melvin, and one finds that the large b case corresponds to the small R and vice versa. Therefore there seems to be a nice correspondence between the shape of the potential and the pattern of the appearance of the tachyons in the heterotic theory; for large R there is a potential barrier to stabilize the vacuum, corresponding to the absence of the tachyon, while for small R no obstruction exists toward the decay, and thus the instability could be already seen in the perturbative level.

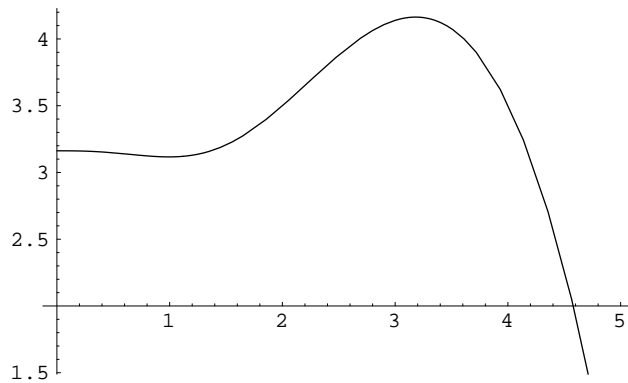


Figure 1: The potential for small b

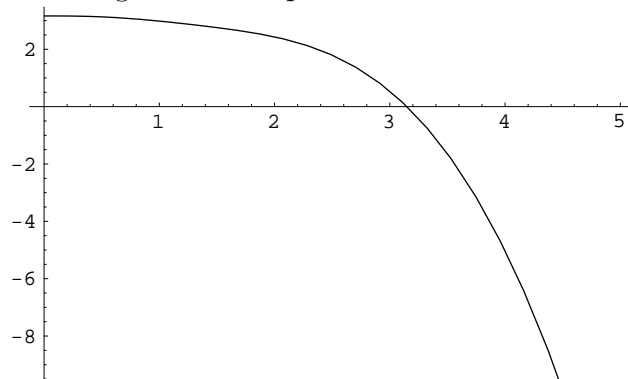


Figure 2: The potential for large b

Now it is easy to deduce the endpoint of the stabilization of the M-theory which would be dual to the heterotic theory on the KK-Melvin. The KK-Melvin would decay into the flat background times S^1 . In view of the Type IIA picture, the R-R flux would be cancelled by the dipoles induced by the spherical D6-branes. Since the energy of the system would decrease by rolling down the potential, the energy of the fluxbrane would also be cancelled. (It is pointed out that such arguments might be subtle [27])

It is natural to expect that the expanding D6-branes will take the D2-D4 systems to infinity since they form bound states. In addition, it would be possible that some of the D2-D4 systems remain and even condense, since the binding energy of them with the D6-branes is finite and they can be separated from the expanding D6-branes. In the latter case, the charges coupled to the gauge fields would condense in the resulting vacuum. We will discuss this situations in the next section.

Recently, it is discussed in [25] that the fluxbrane (4.1) can be constructed from the D6-branes and the anti-D6-branes which are placed far apart from each other. According to this viewpoint, the decay of the fluxbrane is a direct consequence of the annihilation of the D6-branes and the spherical D6-branes. This result would strongly suggest that the endpoint of the decay of the fluxbrane is the supersymmetric vacuum without the R-R flux, as is conjectured in [3]. Thus the stable vacuum of the M-theory discussed above

would be $\mathbf{R}^{1,5} \times K3 \times S^1$ on which M-theory is dual to the heterotic theory on $T^3 \times S^1$.

5 String theory duals

In this section, we consider several heterotic theories and their duals to discuss effects of the charged tachyon condensation on the gauge symmetries. The background of each heterotic theory is of the form $X \times \text{Melvin}$, where X is a compact manifold. We will argue that in some cases the rank of the gauge group is unchanged by the tachyon condensation, as opposed to the naive expectation.

In the following discussions, we will restrict ourselves to the situations in which the gauge charges would condense but the winding number is not. One can consider such special situations since there are tachyons with the winding number both $+1$ and -1 . Therefore, in other words, we will consider the condensation of composite particles which consist of tachyons with total winding number zero. We will comment on more general situations in section 6.

(i) No compact space

Consider the simplest case; the $SO(32)$ heterotic theory on the KK-Melvin with the Wilson line (3.1). Recall that this theory has the gauge group $SO(32)$ and the tachyons in the vector representation of the $SO(32)$. Therefore, if these tachyons acquire nonzero vevs, one may expect that the gauge symmetry is broken down and the rank of the gauge group is reduced.

As discussed in section 2 and in [8], the dual description of this theory would be Type I theory on the KK-Melvin with the corresponding Wilson line. In the Type I picture, the winding strings around the S^1 of the KK-Melvin in the heterotic picture correspond to the winding D-strings around the same S^1 . The vector representation of the $SO(32)$ can be constructed from the open strings with one end on the D-string and the other end on the D9-branes. Since there is no Wilson line of the worldvolume gauge field on the D-string, the fermionic degrees of freedom of the open strings obey the anti-periodic boundary condition along the S^1 . The Wilson line of the spacetime gauge field does not affect the open strings since they have the vector index of the $SO(32)$. Therefore the strings have the mode expansion with the half-integral moding, and the vector states can be obtained. Thus the duality suggests that the states corresponding to the tachyons in the heterotic theory are the winding D-strings with open strings stretched between them and the D9-branes.

Now we can consider the condensation of the tachyonic states. We have assumed that the winding number is totally cancelled. This means that there are the same number of the D-strings and the anti-D-strings. Thus they are pair-annihilated and there remain

the fundamental open strings. However, because of the absence of the D-strings, both ends of the fundamental open strings must be attached to the D9-branes. The resulting strings need not be attached to a single D9-brane, so there are still charged states, but they are now in the adjoint representation of the $SO(32)$. Therefore we conclude that the endpoint of the charged tachyon condensation with no net winding number would be the supersymmetric Type I background with a nonzero vev of adjoint fields. One can easily see that the only adjoint field that can condense is the massless scalar A_y which comes from the y -component of the gauge field in ten dimensions. The generic vev of the A_y breaks the gauge group to $U(1)$ ¹⁶ and the rank remains the same. This result is reasonable since the rank of the gauge group must be sixteen to ensure the anomaly cancellation in ten dimensions in the Type I theory.

The above phenomenon can be understood in terms of the low energy field theory. From the assumption that the net winding number vanishes, the tachyons in the vector representation would form pairs with other tachyons which have the opposite winding number. Then the pairs would be in the adjoint representation. Hence the fields which have nonzero vevs would be in the adjoint, not in the vector. The above Type I picture shows the mechanism explicitly.

(ii) $X = T^2$

By compactifying the heterotic theory discussed above on T^2 , one can find other situations with more general gauge groups and representations of tachyons, depending on the point of the moduli space of the T^2 compactification. The dual theory would be F-theory [41] compactified on $K3 \times \text{Melvin}$.²

Recall the duality relation between F-theory on $K3$ and string theories in eight dimensions. If the $K3$ is the orbifold T^4/\mathbf{Z}_2 , F-theory compactified on the $K3$ can be described by a perturbative Type IIB orientifold [42]. In this picture, Type IIB theory is compactified on T^2/\mathbf{Z}_2 , and one orientifold plane and four D7-branes exist on each fixed point. This orientifold is just the T-dual of Type I theory compactified on T^2 with the Wilson lines

$$A_5^I R_5 = \left(\frac{1}{2}, \frac{1}{2}, 0^4, 0^4 \right), \quad (5.1)$$

$$A_6^I R_6 = \left(\frac{1}{2}, 0^4, \frac{1}{2}, 0^4 \right), \quad (5.2)$$

where x^5, x^6 are the coordinates of the T^2 . This theory is also dual to the heterotic theory on T^2 .

Consider the heterotic theory on $T^2 \times \text{Melvin}$ with the above Wilson lines. The gauge group of the theory is $SO(8)^4$. For each $SO(8)$, there are tachyons in the vector representation. The dual theory would be the Type IIB orientifold whose target space is $T^2/\mathbf{Z}_2 \times \text{Melvin}$. The states corresponding to the heterotic tachyons can be deduced as follows. The winding strings around the S^1 of the KK-Melvin are mapped to D3-branes wrapped on the same S^1 and the T^2/\mathbf{Z}_2 . The D3-branes can have the charges for the

²I would like to thank T.Tani for pointing out this dual picture.

gauge fields on the D7-branes, which come from the open strings stretched between them and the D7-branes, as in the previous case. One can show that the quantization of the strings produces the vector states. Hence we could expect that these D3-brane states are the counterpart of the heterotic tachyons.

The condensation of such states can be discussed in the similar manner to the previous case. By assumption, the D3-branes are completely pair-annihilated, and the remaining open strings must be attached to the D7-branes at the fixed points. Therefore, in the end some adjoint fields living on the D7-branes would acquire nonzero vevs. The candidates for the adjoint fields are three massless scalars A_i on the D7-branes. The generic vevs of the three adjoint fields can break the gauge symmetry completely. However, all the vevs does not correspond to the vacuum configurations. Since it is expected that there are sixteen supercharges at the endpoint of the decay, the potential for A_i is completely determined

$$V = -\frac{1}{4}\text{Tr}[A_i, A_j]^2. \quad (5.3)$$

To obtain the vacuum configuration, all A_i 's must commute with each other. Then the gauge group is in general broken to its maximal Abelian subgroup, i.e. the rank is preserved.

In general, the dual of the heterotic theory on $T^2 \times \text{Melvin}$ would be F-theory compactified on $K3 \times \text{Melvin}$. The K3 is an elliptic fibration over \mathbf{P}^1 . This theory would have a description in terms of Type IIB theory, although it is not a perturbative one in general. The target space is the KK-Melvin times the base space \mathbf{P}^1 of the K3, and the modulus of the fiber corresponds to the linear combination of the dilaton and the R-R 0-form at each point of the base space. The fiber degenerates at 24 points at which there is a D7-brane (or in general a (p,q) 7-brane).

The winding number in the heterotic theory would be mapped to D3-branes wrapped on the $\mathbf{P}^1 \times S^1$. This is true at the orientifold limit discussed above, since T^2/\mathbf{Z}_2 is topologically equivalent to \mathbf{P}^1 , and this correspondence would still hold after the background is deformed continuously. The heterotic tachyons would correspond to the D3-branes with open (p,q)-strings, in general, and after the annihilation of the D3-branes the open strings are attached to the 7-branes. Such string configuration produces adjoint representations of the gauge groups [43].

(iii) $X = T^4$

It seems interesting to consider this case, since in the dual picture the condensation of D-branes should occur. The condensation of D-branes have been discussed in the context of topology-changing transitions of the target space, for example, the conifold transitions [44].

The dual theory would be Type IIA theory on $K3 \times \text{Melvin}$. We are interested in situations in which there are non-Abelian gauge symmetries, and hence we consider a singular K3. The winding number would be mapped to NS5-branes wrapped on the K3 and the S^1 of the KK-Melvin. In the Type IIA picture, the spacetime gauge field

comes from the R-R fields, and hence the gauge charges would be provided by D2-branes wrapped on 2-cycles of the K3. Thus the corresponding states to the heterotic tachyons would be bound states of the NS5-branes and the D2-branes.

The condensation of the charges corresponds to the condensation of the D2-branes. In order for the charged states to have a nonzero vev, there must be massless states coming from the D2-branes. This can be realized when the D2-branes are wrapped on vanishing cycles at a singularity of the K3. The massless states are the vector multiplet. Thus it is the adjoint states that acquire nonzero vevs, as in the previous cases. The supersymmetry at the endpoint of the decay prevent the rank of the gauge group from reducing.

$$(iv) \ X = K3 \times T^2$$

We have discussed the situations which are expected to have sixteen supercharges at the end of the condensation, and thus what happens after the charged tachyon condensation is severely restricted by the supersymmetry. Therefore, it would be natural to expect that something different may occur if there are less number of supercharges recovered after the condensation.

In this case, the dual theory would be Type IIA theory on a Calabi-Yau threefold times the KK-Melvin. The threefold is a K3 fibration over \mathbf{P}^1 . Non-Abelian gauge symmetries appear when all the fibers become singular simultaneously [45]. In general, the condensation of charged states breaks the gauge symmetry and often reduces the rank. Interestingly enough, it is argued that there are geometric transitions of the threefold [45] which reduce the rank of the gauge group. These transitions are realized as deformations of the complex structure of the singular threefold. The deformations of the complex structure in general remove, in some sense, vanishing cycles, and the homology can change, in contrast to the blowing-up of singularities. Therefore, such deformations would be appropriate to describe the condensation of charged states.

The correspondence between the deformations of the complex structure and relevant deformations of a theory also seems natural from the worldsheet point of view. Consider the string propagation in the vicinity of the singularity in the K3 fiber. This is approximated by the string theory on the ALE space, which has a description in terms of the Landau-Ginzburg model [46]; for A_{n-1} singularity, the superpotential is

$$W(w, x, y, z) = w^{-n} + f(x, y, z), \quad (5.4)$$

$$f(x, y, z) = x^2 + y^2 + z^n, \quad (5.5)$$

where $f(x, y, z) = 0$ defines the A_{n-1} singularity. The deformations of the complex structure correspond to adding polynomials to $f(x, y, z)$, and hence they change the superpotential. On the other hand, the fixed point of the RG flow is determined by the superpotential [47][48]. Therefore, the deformations of $f(x, y, z)$ would change the RG behavior, which is an appropriate property for the relevant deformations of the theory.

It might be possible that the tachyon condensation for $X = T^4$ case could be understood in terms of the background geometry in the dual Type IIA picture. For the K3 manifold, the deformations of the complex structure are equivalent to the deformations

of the Kähler structure. Thus all deformations of singularities can be reinterpreted as the blowing-up, and hence the rank of the gauge symmetry in the Type IIA picture is not reduced.

6 Discussion

We have discussed the charged tachyon condensation in the heterotic theory on the KK-Melvin, and focused on whether (and how) the gauge symmetry breaking occurs. The discussions are based on the assumption that the adiabatic argument can be applied to our non-supersymmetric setting. Once we assume that it is the case, we can argue the charged tachyon condensation in terms of the various dual pictures. The gravitational dual strongly suggests that the supersymmetry-breaking background would decay into a supersymmetric vacuum, and the gauge charges could condense. To see the effect of the condensation of the charges on the gauge symmetry, we have investigated string theory duals and argue the fate of the gauge group, in particular of its rank.

We have excluded the situations in which there is a net winding number after the condensation. One could argue that the investigations of such situations are not necessary as follows: After the condensation, any winding states would turn into massive states. It can be deduced from the fact that in every dual picture the winding number is mapped to a number of an extended object with finite volume. It might be possible for the winding states to be massless if the corresponding S^1 shrinks down (or decompactifies in the T-dual picture) after the condensation. However, it is suggested that the condensation deforms the background in the opposite direction [18][25]. Since massive states would not have nonzero vevs, the condensation with nonvanishing winding number would not occur.

We have not discussed any quantitative properties of the condensation. It is mainly because there is no preserved supersymmetry at all, during the condensation. Recently, it is pointed out that the worldsheet supersymmetry is a powerful tool to consider closed string tachyons in non-supersymmetric backgrounds [29][30][27], and moreover, the spacetime action of the tachyons are proposed [31]. It is very interesting to apply such techniques to the heterotic cases. Note that in such research the perturbations which preserve the worldsheet supersymmetry is considered. However, it would be important to discuss more general perturbations, although it is much harder to handle.

An interesting viewpoint obtained in our analysis is that the tachyon condensation in the heterotic theory might correspond to a geometric deformation of the background in the dual Type II theory. This would imply that the IR dynamics of such a theory might be quite different from its UV dynamics. It will be interesting to construct such examples explicitly.

In the case of the open string tachyon condensation, the phenomena can be interpreted as decays of some unstable D-brane systems. It seems that in the case of closed string

tachyon condensation there is no such a unified understanding of the phenomena. Fortunately, many non-supersymmetric backgrounds are known to have tachyons, and hence it will be important to understand them in a unified way.

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